

Simulink[®] Control Design[™]

Getting Started Guide



MATLAB[®]&SIMULINK[®]

R2016a



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Simulink® Control Design™ Getting Started Guide

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March 2016	Online only	Revised for Version 4.3 (Release 2016a)

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Product Overview

Simulink Control Design Product Description

Linearize models and design control systems

Simulink® Control Design™ lets you design and analyze control systems modeled in Simulink. You can automatically tune PID controller gains to meet performance requirements. You can also automatically tune arbitrary SISO and MIMO control architectures. You can find operating points and compute exact linearizations of Simulink models at various operating conditions. Simulink Control Design provides tools that let you compute simulation-based frequency responses without modifying your model.

Key Features

- Automatic tuning of PID, gain-scheduled, and arbitrary SISO and MIMO control systems
- Operating-point calculation (trimming) and linearization of models
- Frequency response estimation from simulation data
- Batch linearization for varying parameters and operating points
- Numerical optimization of compensators to meet time-domain and frequency-domain requirements (with Simulink Design Optimization™)

Steady-State Operating Points

- “What Is a Steady-State Operating Point?” on page 2-2
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What Is a Steady-State Operating Point?

A *steady-state operating point* of a model, also called an equilibrium or *trim* condition, includes state variables that do not change with time.

A model might have several steady-state operating points. For example, a hanging pendulum has two steady-state operating points. A *stable steady-state operating point* occurs when a pendulum hangs straight down. That is, the pendulum position does not change with time. When the pendulum position deviates slightly, the pendulum always returns to equilibrium; small changes in the operating point do not cause the system to leave the region of good approximation around the equilibrium value.

An *unstable steady-state operating point* occurs when a pendulum points upward. As long as the pendulum points *exactly* upward, it remains in equilibrium. However, when the pendulum deviates slightly from this position, it swings downward and the operating point leaves the region around the equilibrium value.

When using optimization search to compute operating points for a nonlinear system, your initial guesses for the states and input levels must be in the neighborhood of the desired operating point to ensure convergence.

When linearizing a model with multiple steady-state operating points, it is important to have the right operating point. For example, linearizing a pendulum model around the stable steady-state operating point produces a stable linear model, whereas linearizing around the unstable steady-state operating point produces an unstable linear model.

Examples and How To

- “Steady-State Operating Points (Trimming) From Specifications” on page 2-3
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More About

- “Computing Steady-State Operating Points”
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Steady-State Operating Points (Trimming) From Specifications

This example shows how to compute a steady-state operating point, or equilibrium operating point, by specifying known (fixed) equilibrium states and minimum state values.

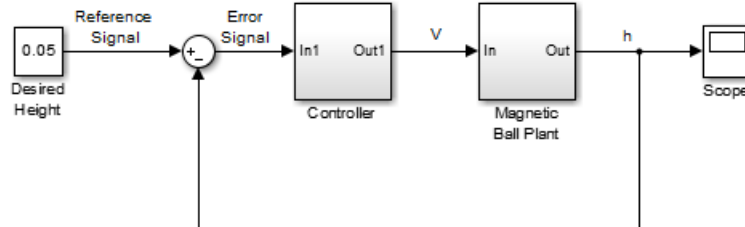
This example finds an operating point of a magnetic ball model at which the height of a levitating magnetic ball remains stable at a desired height of 0.05 m.

Code Alternative

Use `findop` to find operating point from specifications. For examples and additional information, see the `findop` reference page. Finding a steady-state operating point is also known as *trimming*.

- 1 Open the Simulink model.

```
sys = 'magball';
open_system(sys)
```



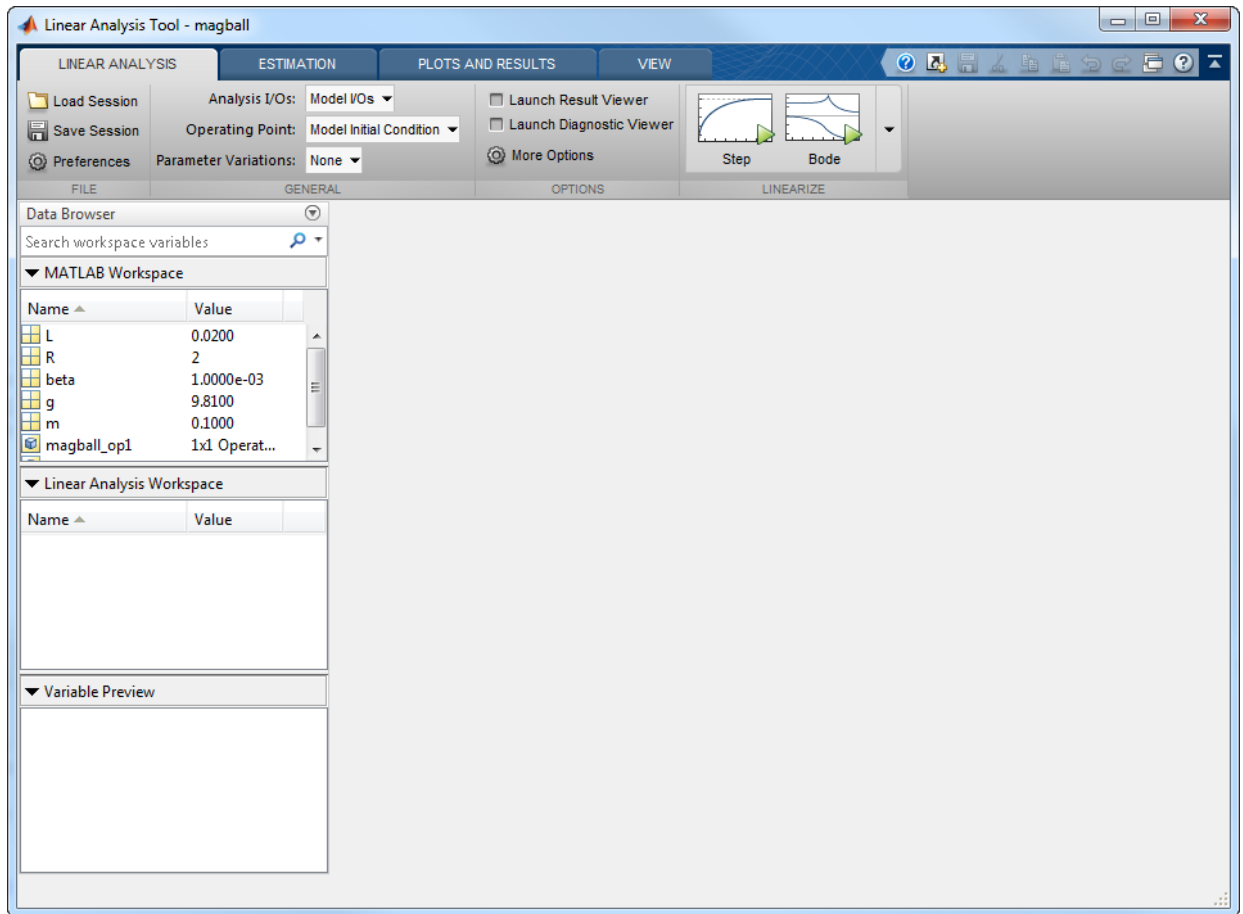
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In this model, the height of the magnetic ball is represent by the plant output, h . Trim the model to find a steady state operating point at which $h = 0.05$.

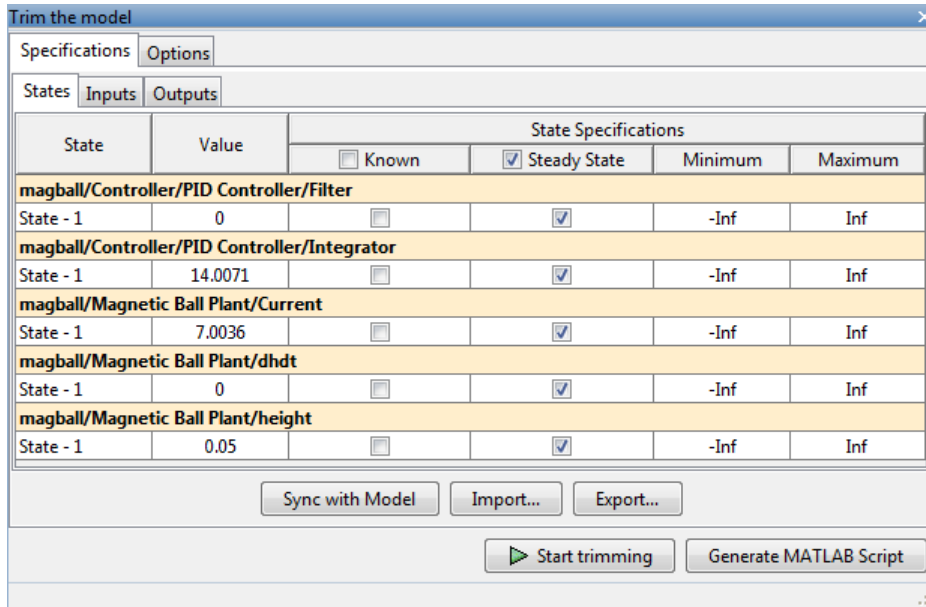
- 2 In the Simulink Editor, select **Analysis > Control Design > Linear Analysis**.

The Linear Analysis Tool for the model opens.

2 Steady-State Operating Points



- 3 In the Linear Analysis Tool, in the **Operating Point** drop-down list, select Trim Model.

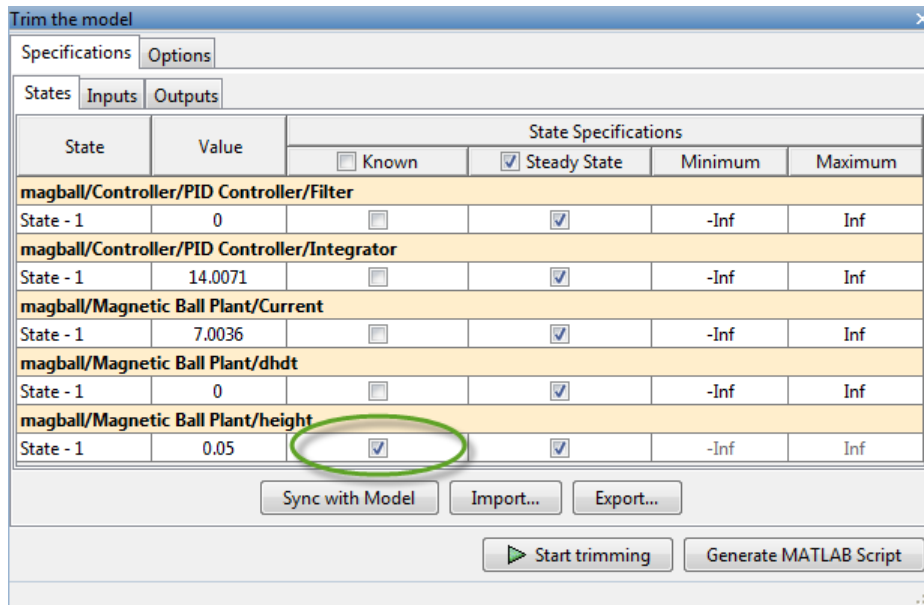


By default, in the **States** tab, the software specifies all model states to be at equilibrium, as shown by the check marks in the **Steady State** column. The **Inputs** and **Outputs** tabs are empty because this model does not have root-level input and output ports.

- 4 Specify a fixed height for the magnetic ball.

In the **States** tab, select **Known** for the **height** state.

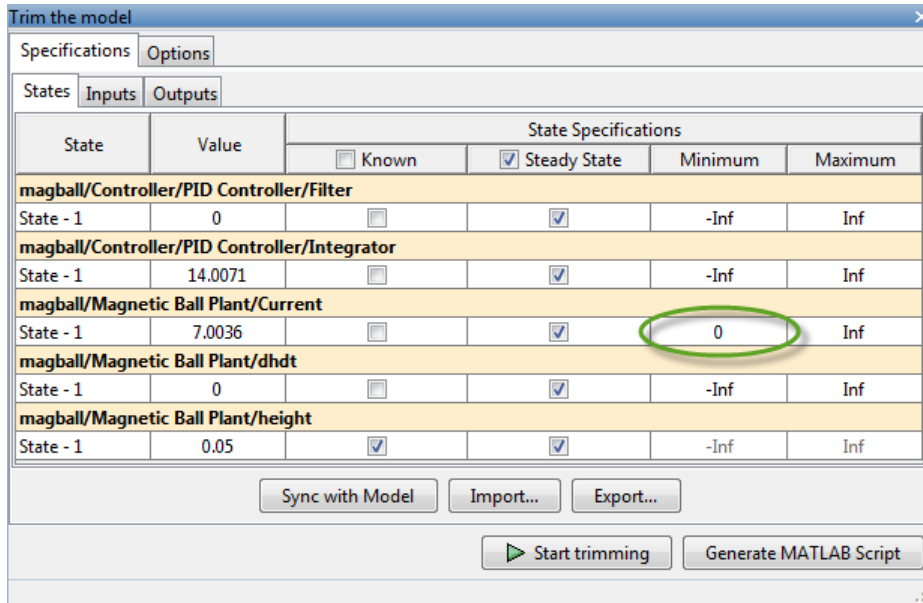
2 Steady-State Operating Points



The height of the ball matches the reference signal height (specified in the **Desired Height** block as 0.05). Since it is known value, the height remains fixed during optimization.

- 5 Limit the plant current to positive values.

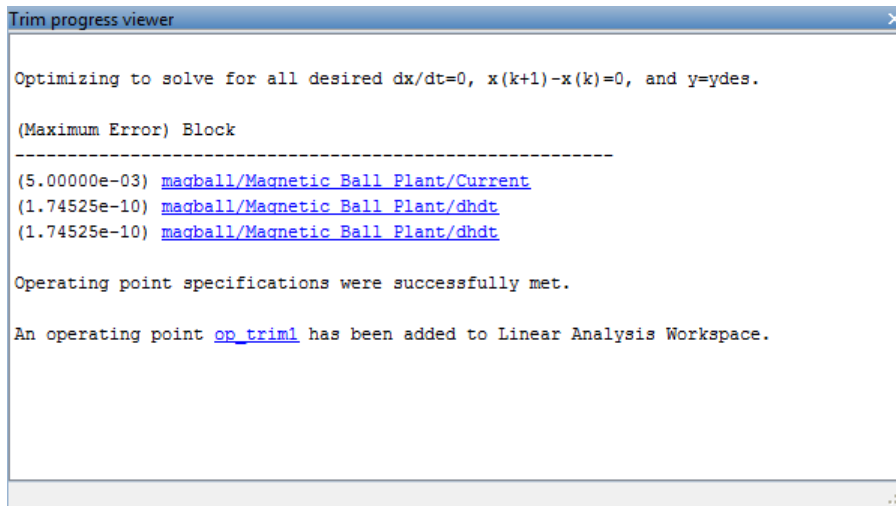
Enter 0 for the **Minimum** bound of the **Current** state.



Since a positive current is required to raise the height of the ball, setting the lower bound to 0 limits the optimization solution to the plant operating range.

- 6 Click **Start trimming** to compute the operating point.

The software uses numerical optimization to find the operating point that meets your specifications.



```
Trim progress viewer
Optimizing to solve for all desired dx/dt=0, x(k+1)-x(k)=0, and y=ydes.

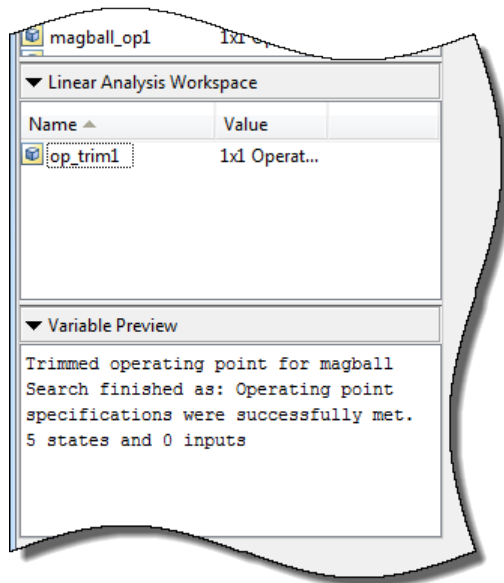
(Maximum Error) Block
-----
(5.00000e-03) magball/Magnetic Ball Plant/Current
(1.74525e-10) magball/Magnetic Ball Plant/dhdt
(1.74525e-10) magball/Magnetic Ball Plant/dhdt

Operating point specifications were successfully met.

An operating point op_trim1 has been added to Linear Analysis Workspace.
```

The Trim progress viewer shows that the optimization algorithm terminated successfully. The (Maximum Error) Block area shows the progress of reducing the error of a specific state or output during the optimization.

A new variable, `op_trim1`, appears in the **Linear Analysis Workspace**.



- 7 Double-click `op_trim1` in the **Linear Analysis Workspace** to evaluate whether the resulting operating point values meet the specifications.

The 'Edit: op_trim1' dialog box displays the 'Optimizer Output' tab. The table below shows the results of the optimization process, with green circles highlighting the 'Actual Value' and 'Actual dx' columns for the 'height' state.

State	Input	Output	Desired Value	Actual Value	Desired dx	Actual dx
magball/Controller/PID Controller/Filter						
State - 1			[-Inf, Inf]	0	0	0
magball/Controller/PID Controller/Integrator						
State - 1			[-Inf, Inf]	14.0071	0	0
magball/Magnetic Ball Plant/Current						
State - 1			[-Inf, Inf]	7.0036	0	4.2064e-11
magball/Magnetic Ball Plant/dhdt						
State - 1			[-Inf, Inf]	0	0	-1.7453e-10
magball/Magnetic Ball Plant/height						
State - 1			0.05	0.05	0	0

An 'Initialize model...' button is located at the bottom right of the dialog box.

In the **State** tab, the **Actual Value** for each state falls within the **Desired Value** bounds. The actual height of the ball is 0.05 m, as specified.

The **Actual dx** column shows the rates of change of the state values at the operating point. Since these values are at or near zero the states are not changing, showing that the operating point is in a steady state.

Related Examples

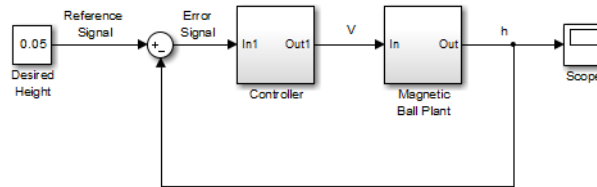
- “Steady-State Operating Points (Trimming) From Specifications” on page 2-3
- “Compute Operating Points at Simulation Snapshots”

More About

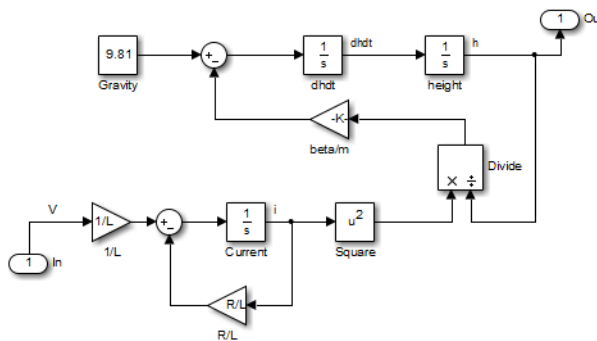
- “magball Simulink Model” on page 2-11
- “Computing Steady-State Operating Points”

magball Simulink Model

The Simulink model magball includes the nonlinear Magnetic Ball Plant in a single-loop feedback system.

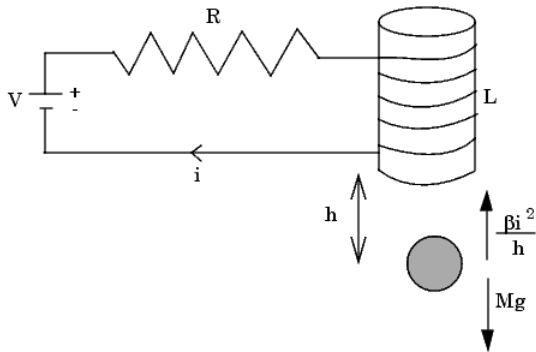


The Magnetic Ball Plant subsystem is shown in the following figure.



The Magnetic Ball Plant model represents an iron ball of mass M . This ball moves under the influence of the gravitational force, Mg , and an induced magnetic force, $\frac{\beta i^2}{h}$. The presence of the squared term in the induced magnetic force results in a nonlinear plant.

The inductor in the electric circuit, shown in the following figure, causes the induced magnetic force. This circuit also includes a voltage source and a resistor.



The following table describes the variables, parameters, differential equations, states, inputs, and outputs of the Magnetic Ball Plant subsystem.

Variables	<p>h is the height of the ball.</p> <p>i is the current.</p> <p>V is the voltage in the circuit.</p>
Parameters	<p>M is the mass of the ball.</p> <p>g is the gravitational acceleration.</p> <p>β is a constant related to the magnetic force.</p> <p>L is the inductance of the coil.</p> <p>R is the resistance of the circuit.</p>
Differential equations	<p>The height of the ball, h, is described in the following equation:</p> $M \frac{d^2 h}{dt^2} = Mg - \frac{\beta i^2}{h}$ <p>The current in the circuit, i, is described in the following equation:</p> $L \frac{di}{dt} = V - iR$

States	h dh/dt i
Inputs	V
Outputs	h

Examples and How To

“Steady-State Operating Points (Trimming) From Specifications” on page 2-3

Linearization

- “Applications of Linearization” on page 3-2
- “Open-Loop Response of Control System for Stability Margin Analysis” on page 3-3
- “Bode Response of Simulink Model” on page 3-7
- “watertank Simulink Model” on page 3-11

Applications of Linearization

Linearization is useful in model analysis and control design applications. After you linearize a Simulink model at a specific operating point, you can use your linear model to:

- Compute the Bode response of the Simulink model.
- Evaluate loop stability margins by computing open-loop response.
- Obtain linear state-space, transfer-function, or zero-pole-gain representation of the combined Simulink model that contains only linear blocks.
- Analyze and compare plant response near different operating points.
- Design linear controller

Classical control system analysis and design methodologies require linear, time-invariant models. Simulink Control Design automatically linearizes the plant when you tune your compensator. See .

- Analyze closed-loop stability.
- Measure the size of resonances in frequency response by computing closed-loop linear model for control system.
- Generate controllers with reduced sensitivity to parameter variations and modeling errors (requires Robust Control Toolbox™).

Examples and How To

- “Open-Loop Response of Control System for Stability Margin Analysis” on page 3-3
- “Bode Response of Simulink Model” on page 3-7
- “Steady-State Operating Points (Trimming) From Specifications” on page 2-3

More About

“Linearizing Nonlinear Models”

Open-Loop Response of Control System for Stability Margin Analysis

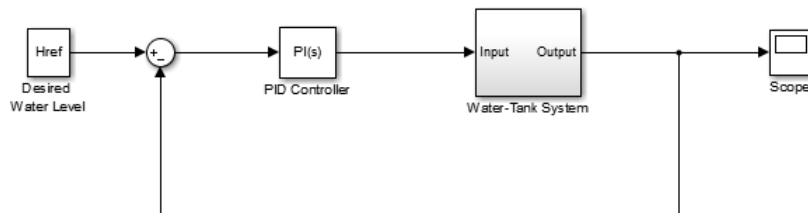
This example shows how to use the Linear Analysis Tool to analyze the open-loop response of a control system.

Compute a linear model of the combined controller-plant system without the effects of the feedback signal. Use a Bode plot of the resulting linear model to see the open-loop response.

1 Open Simulink model.

```
sys = 'watertank';
open_system(sys)
```

The Water-Tank System block represents the plant in this control system and contains all of the system nonlinearities.

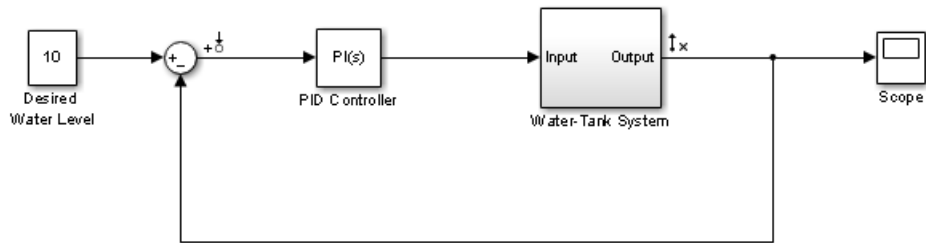


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2 In the Simulink Editor, define the portion of the model to linearize:

- a Right-click the PID Controller block input signal (the output of the Sum block). Select **Linear Analysis Points > Input Perturbation**.
- b Right-click the Water-Tank System output signal, and select **Linear Analysis Points > Open-loop Output**.

Annotations appear in the model indicating which signals are designated as linearization I/O points.

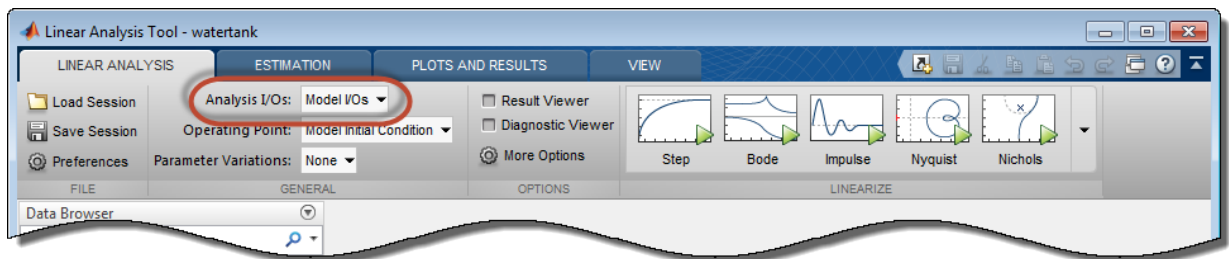


Tip Alternatively, if you do not want to introduce changes to the Simulink model, you can specify the linearization I/O points in the Linear Analysis Tool. See “Specify Portion of Model to Linearize in Linear Analysis Tool”.

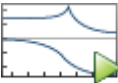
- 3 Open the Linear Analysis Tool for the model.

In the Simulink Editor, select **Analysis > Control Design > Linear Analysis**.

By default, the I/O points you specified in the model are the selected Analysis I/Os for linearization, as displayed in the **Analysis I/Os** menu.



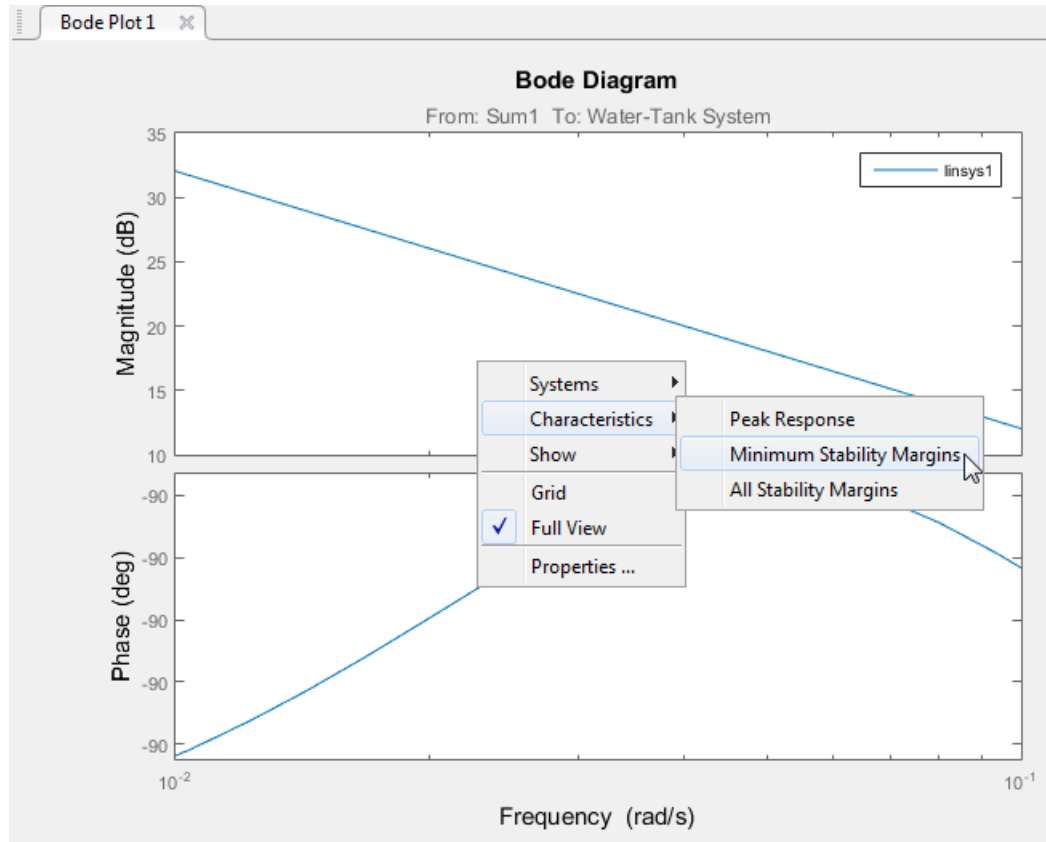
- 4 Linearize the model with the specified I/Os, and generate a Bode plot of the linearized model.

Click  **Bode**. The Bode plot of the linearized plant appears.

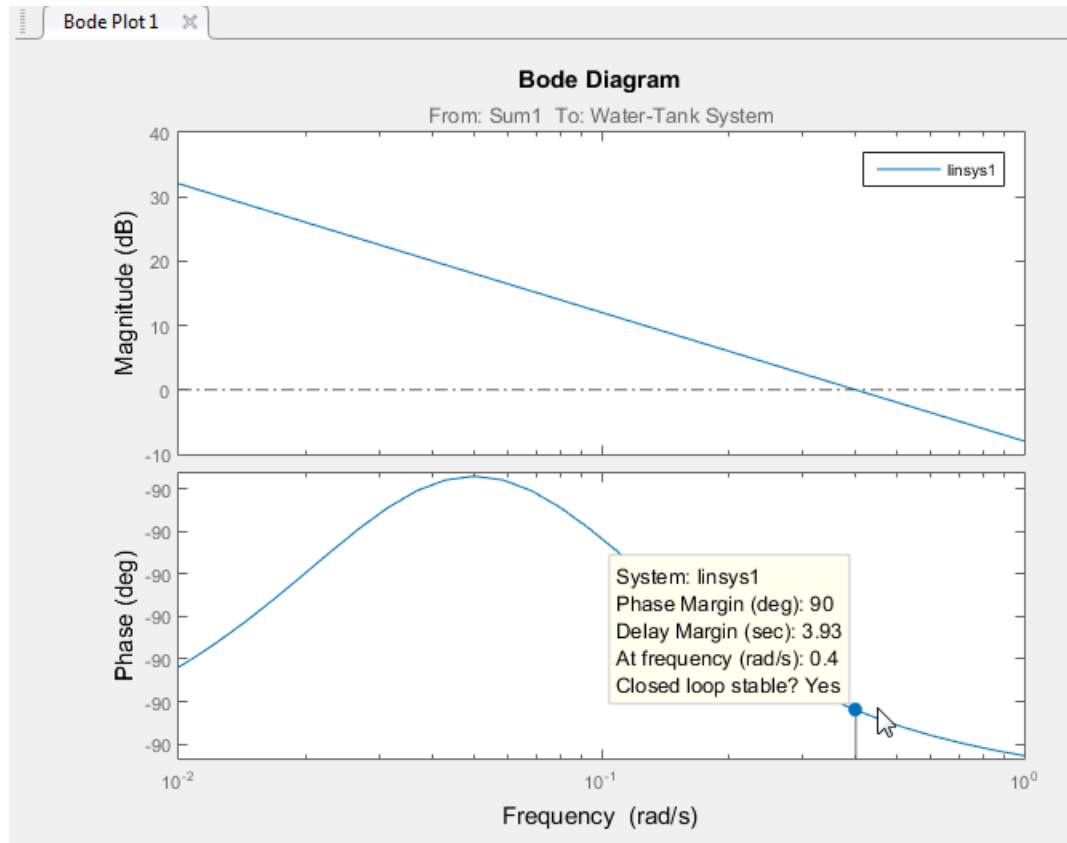
Tip Instead of a Bode plot, generate other response types by clicking the corresponding button in the plot gallery.

- 5 View the minimum stability margins for the model.

Right-click the plot and select **Characteristics > Minimum Stability Margins**.



The Bode plot displays the phase margin marker. Click the marker to show a data tip that contains the phase margin value.



6 Close Simulink model.

```
bdclose(sys);
```

Related Examples

- “Bode Response of Simulink Model” on page 3-7
- “Steady-State Operating Points (Trimming) From Specifications” on page 2-3

More About

- “Linearizing Nonlinear Models”
- “watertank Simulink Model” on page 3-11

Bode Response of Simulink Model

This example shows how to use the Linear Analysis Tool to linearize a model at the operating point specified in the model. The model operating point consists of the model initial state values and input signals.

The Linear Analysis Tool linearizes at the model operating point by default. If you want to specify a different operating point for linearization, see “Linearize at Trimmed Operating Point”.

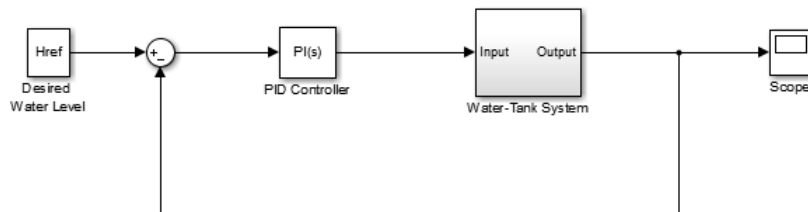
Code Alternative

Use `linearize`. For examples and additional information, see the `linearize` reference page.

- 1 Open Simulink model.

```
sys = 'watertank';
open_system(sys)
```

The Water-Tank System block represents the plant in this control system and includes all of the system nonlinearities.



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- 2 Open the Linear Analysis Tool for the model.

In the Simulink Editor, select **Analysis > Control Design > Linear Analysis**.

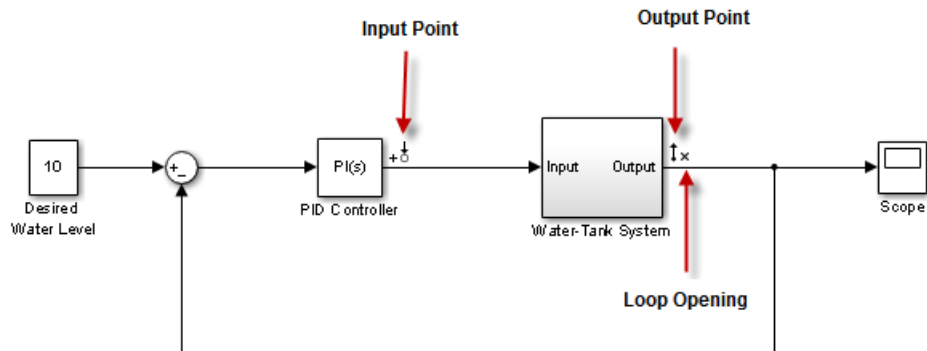
- 3 In the Simulink Editor, define the portion of the model to linearize:

- a Right-click the PID Controller block output signal, which is the input to the plant. Select **Linear Analysis Points > Input Perturbation**.

- b** Right-click the Water-Tank System output signal, and select **Linear Analysis Points > Open-loop Output**.

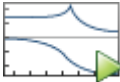
Inserting this open loop point removes the effects of the feedback signal on the linearization without changing the model operating point.

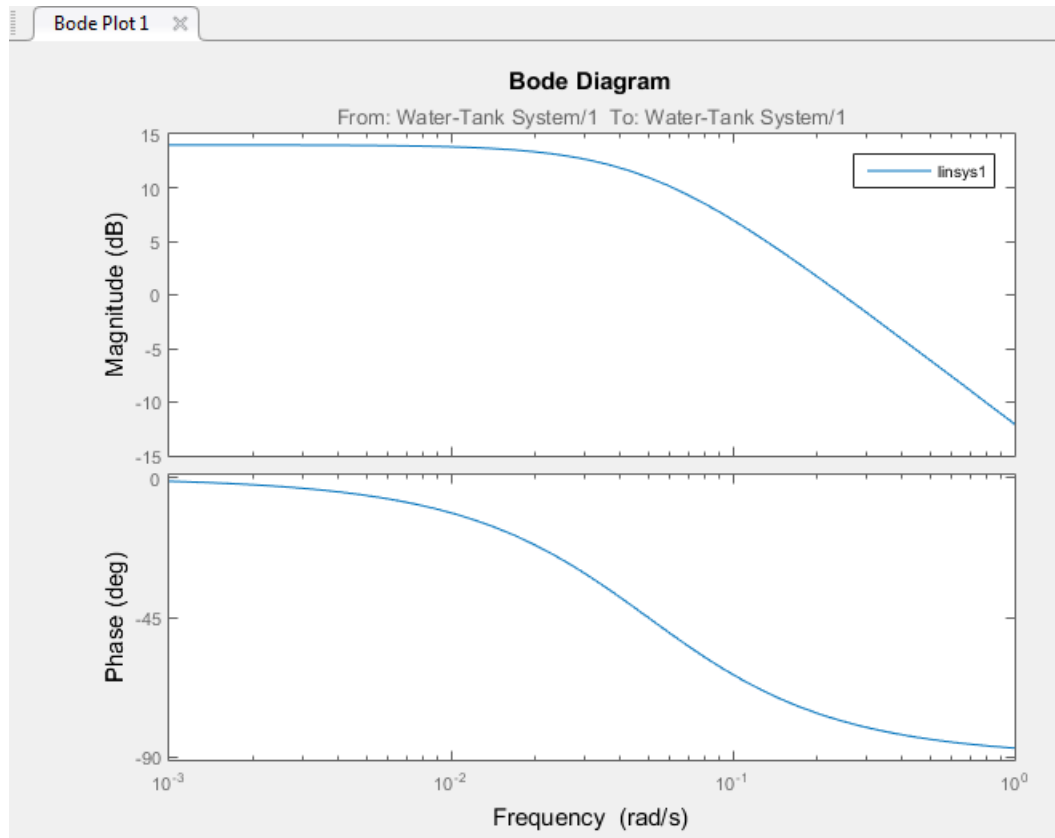
When you add linear analysis points, marker appear at their locations in the model.



Tip Alternatively, if you do not want to introduce changes to the Simulink model, you can specify the linearization I/O points in the Linear Analysis Tool. See “Specify Portion of Model to Linearize in Linear Analysis Tool”.

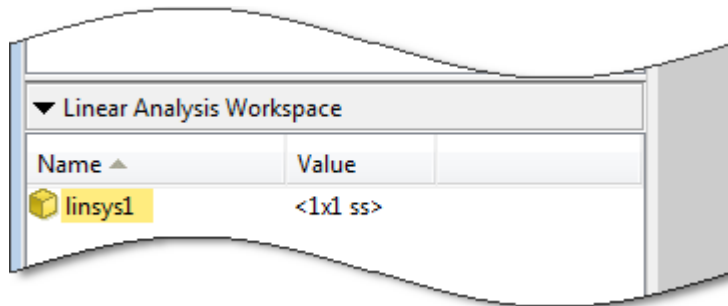
- 4** Linearize the model with the specified I/Os, and generate a Bode plot of the linearized model.

Click  **Bode**. The Bode plot of the linearized plant appears.



Tip Instead of a Bode plot, generate other response types by clicking the corresponding button in the plot gallery.

The linearized system, `linsys1`, appears in the Linear Analysis Workspace.



`linsys1` represents the system linearized at the model operating point. If you do not specify an operating point for linearization, the Linear Analysis Tool uses the model operating point by default.

- 5 Close Simulink model.

```
bdclose(sys);
```

Related Examples

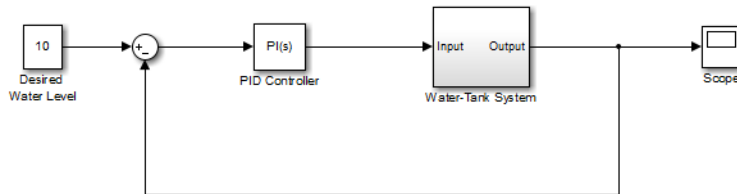
- “Open-Loop Response of Control System for Stability Margin Analysis” on page 3-3
- “Steady-State Operating Points (Trimming) From Specifications” on page 2-3

More About

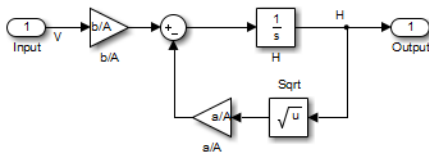
- “Linearizing Nonlinear Models”
- “watertank Simulink Model” on page 3-11

watertank Simulink Model

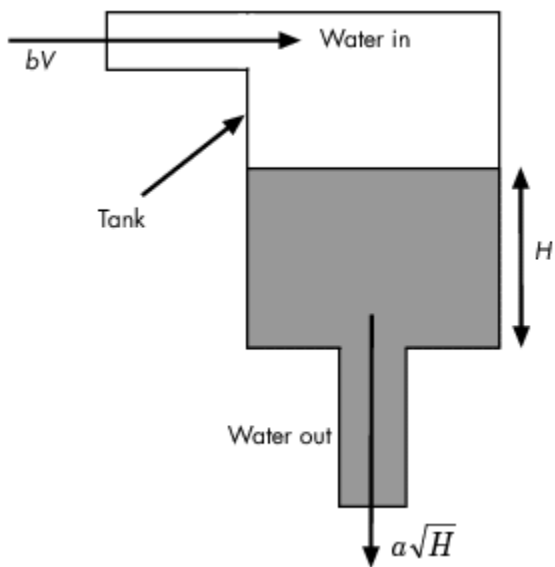
The Simulink model `watertank` includes the nonlinear Water-Tank System plant and a PI controller in a single-loop feedback system.



The Water-Tank System is shown in the following figure.



Water enters the tank from the top at a rate proportional to the voltage, V , applied to the pump. The water leaves through an opening in the tank base at a rate that is proportional to the square root of the water height, H , in the tank. The presence of the square root in the water flow rate results in a nonlinear plant.



The following table describes the variables, parameters, differential equations, states, inputs, and outputs of the Water-Tank System.

Variables	<p>H is the height of water in the tank.</p> <p>Vol is the volume of water in the tank.</p> <p>V is the voltage applied to the pump.</p>
Parameters	<p>A is the cross-sectional area of the tank.</p> <p>b is a constant related to the flow rate into the tank.</p> <p>a is a constant related to the flow rate out of the tank.</p>
Differential equation	$\frac{d}{dt} Vol = A \frac{dH}{dt} = bV - a\sqrt{H}$
States	H
Inputs	V

Outputs

H

PID Control Design

PID Controller Tuning in Simulink

This example shows how to automatically tune a PID Controller block using PID Tuner.

Introduction of the PID Tuner

PID Tuner provides a fast and widely applicable single-loop PID tuning method for the Simulink® PID Controller blocks. With this method, you can tune PID controller parameters to achieve a robust design with the desired response time.

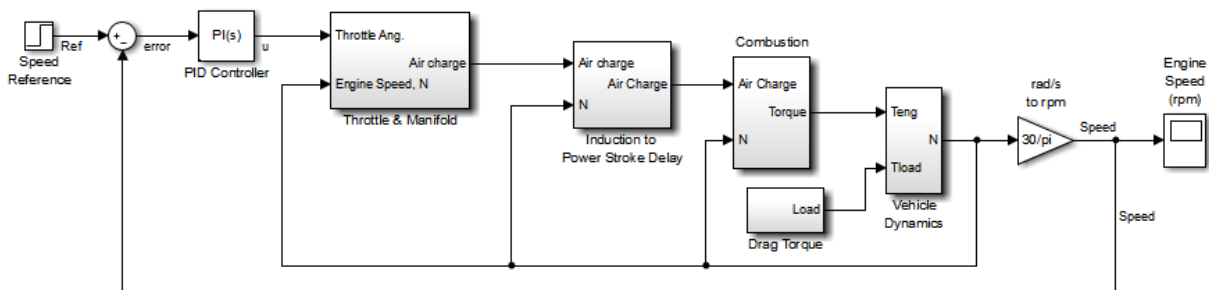
A typical design workflow with the PID Tuner involves the following tasks:

- (1) Launch the PID Tuner. When launching, the software automatically computes a linear plant model from the Simulink model and designs an initial controller.
- (2) Tune the controller in the PID Tuner by manually adjusting design criteria in two design modes. The tuner computes PID parameters that robustly stabilize the system.
- (3) Export the parameters of the designed controller back to the PID Controller block and verify controller performance in Simulink.

Opening the Model

Open the engine speed control model with PID Controller block and take a few moments to explore it.

```
open_system('scdspeedctrlpidblock');
```



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Design Overview

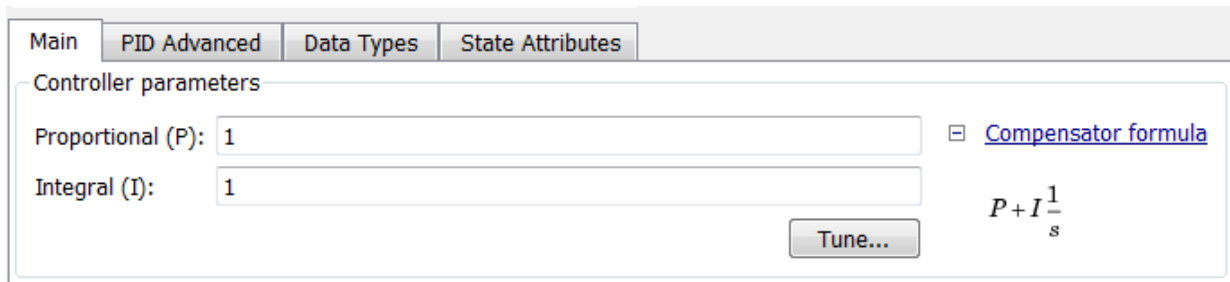
In this example, you design a PI controller in an engine speed control loop. The goal of the design is to track the reference signal from a Simulink step block `scdspeedctrlpidblock/Speed Reference`. The design requirements are:

- Settling time under 5 seconds
- Zero steady-state error to the step reference input.

In this example, you stabilize the feedback loop and achieve good reference tracking performance by designing the PI controller `scdspeedctrl/PID Controller` in the PID Tuner.

Opening the PID Tuner

To launch the PID Tuner, double-click the PID Controller block to open its block dialog. In the **Main** tab, click **Tune**.

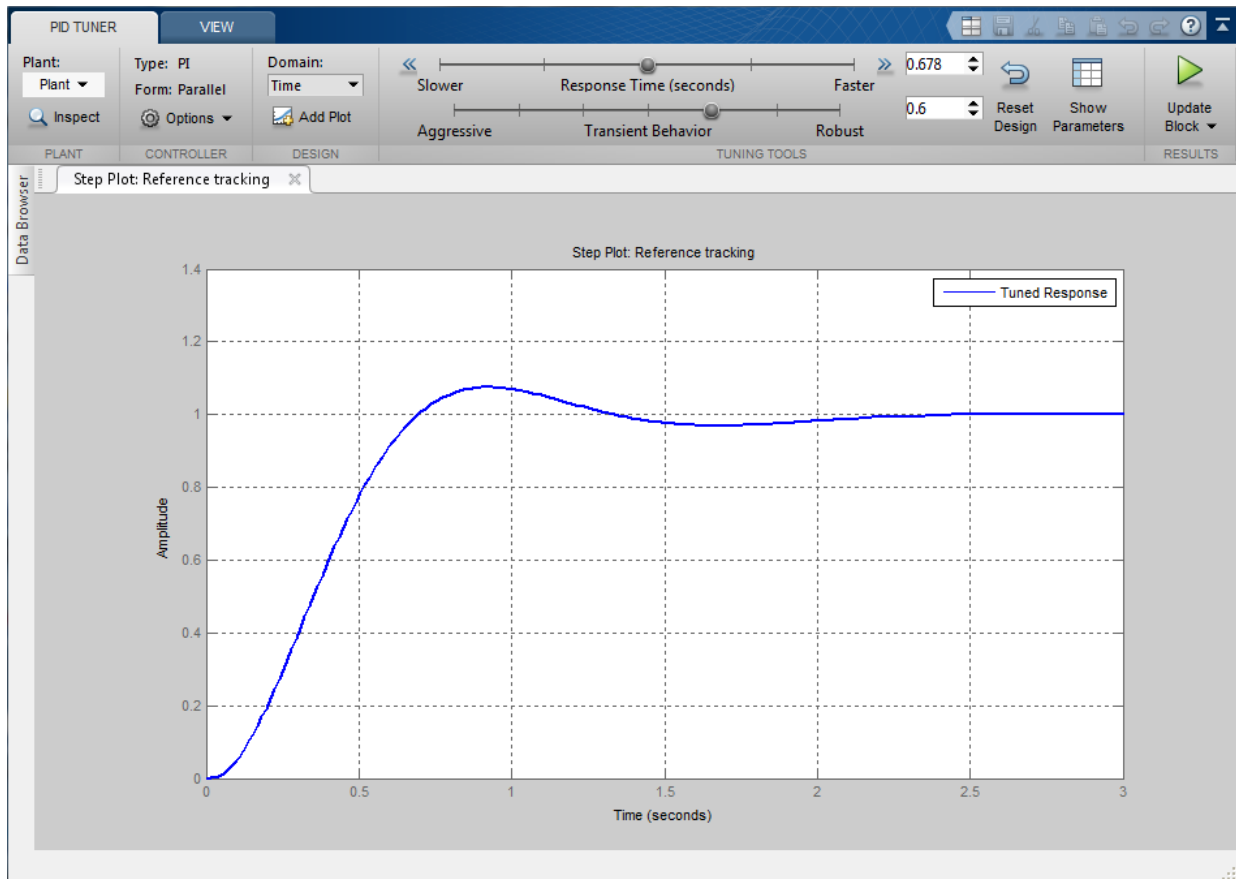


Initial PID Design

When the PID Tuner launches, the software computes a linearized plant model seen by the controller. The software automatically identifies the plant input and output, and uses the current operating point for the linearization. The plant can have any order and can have time delays.

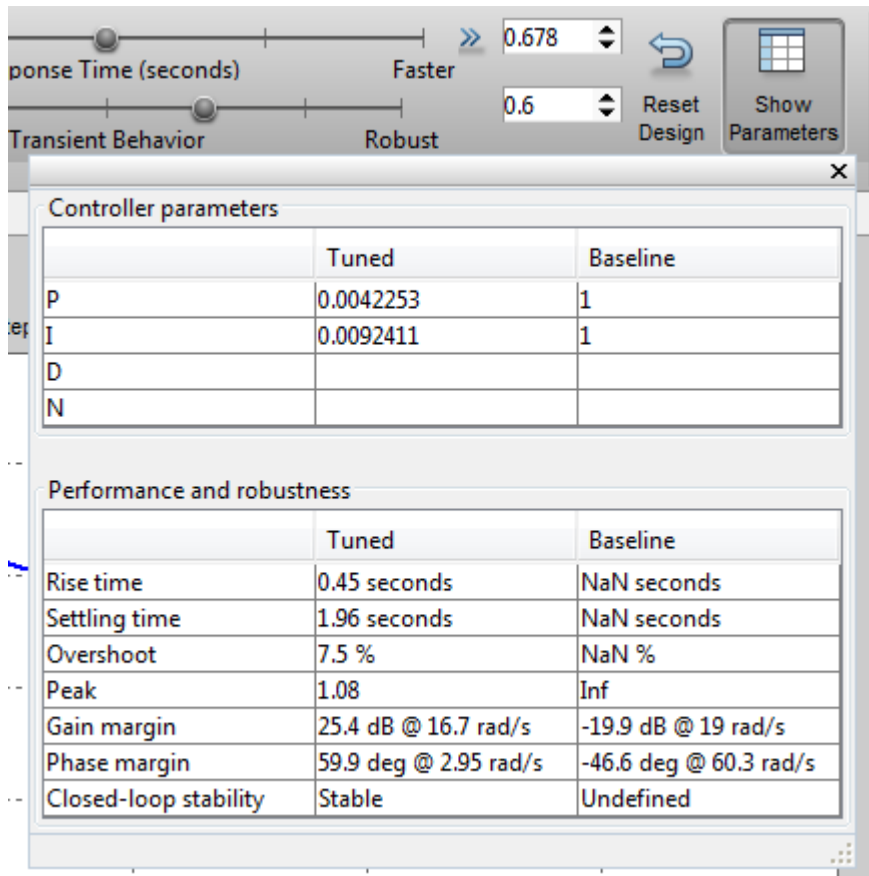
The PID Tuner computes an initial PI controller to achieve a reasonable tradeoff between performance and robustness. By default, step reference tracking performance displays in the plot.

The following figure shows the PID Tuner dialog with the initial design:



Displaying PID Parameters

Click **Show parameters** to view controller parameters P and I, and a set of performance and robustness measurements. In this example, the initial PI controller design gives a settling time of 2 seconds, which meets the requirement.

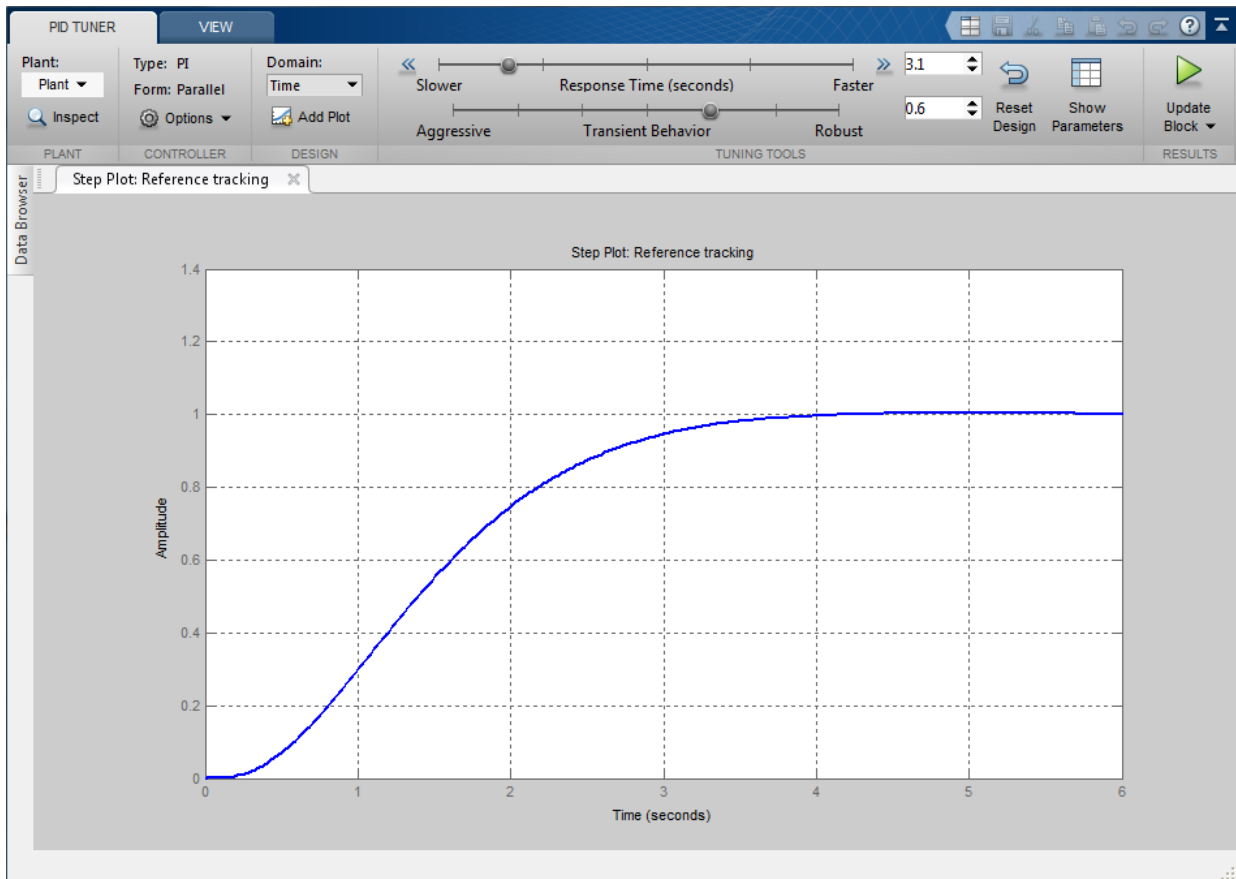


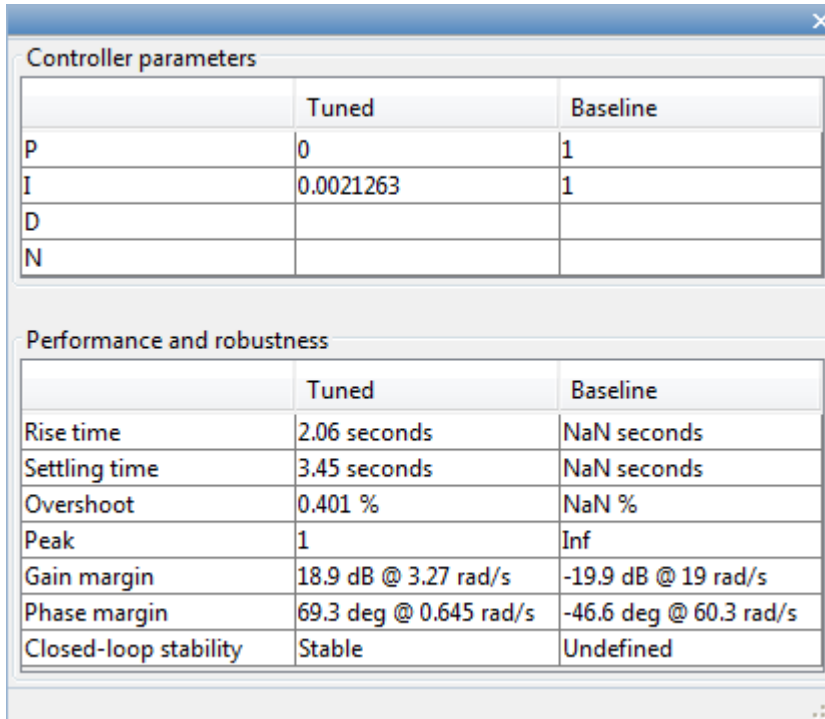
Adjusting PID Design in the PID Tuner

The overshoot of the reference tracking response is about 7.5 percent. Since we still have some room before reaching the settling time limit, you could reduce the overshoot by increasing the response time. Move the response time slider to the left to increase the closed loop response time. Notice that when you adjust response time, the response plot and the controller parameters and performance measurements update.

The following figure shows an adjusted PID design with an overshoot of zero and a settling time of 4 seconds. The designed controller effectively becomes an integral-only controller.

4 PID Control Design





Controller parameters		
	Tuned	Baseline
P	0	1
I	0.0021263	1
D		
N		

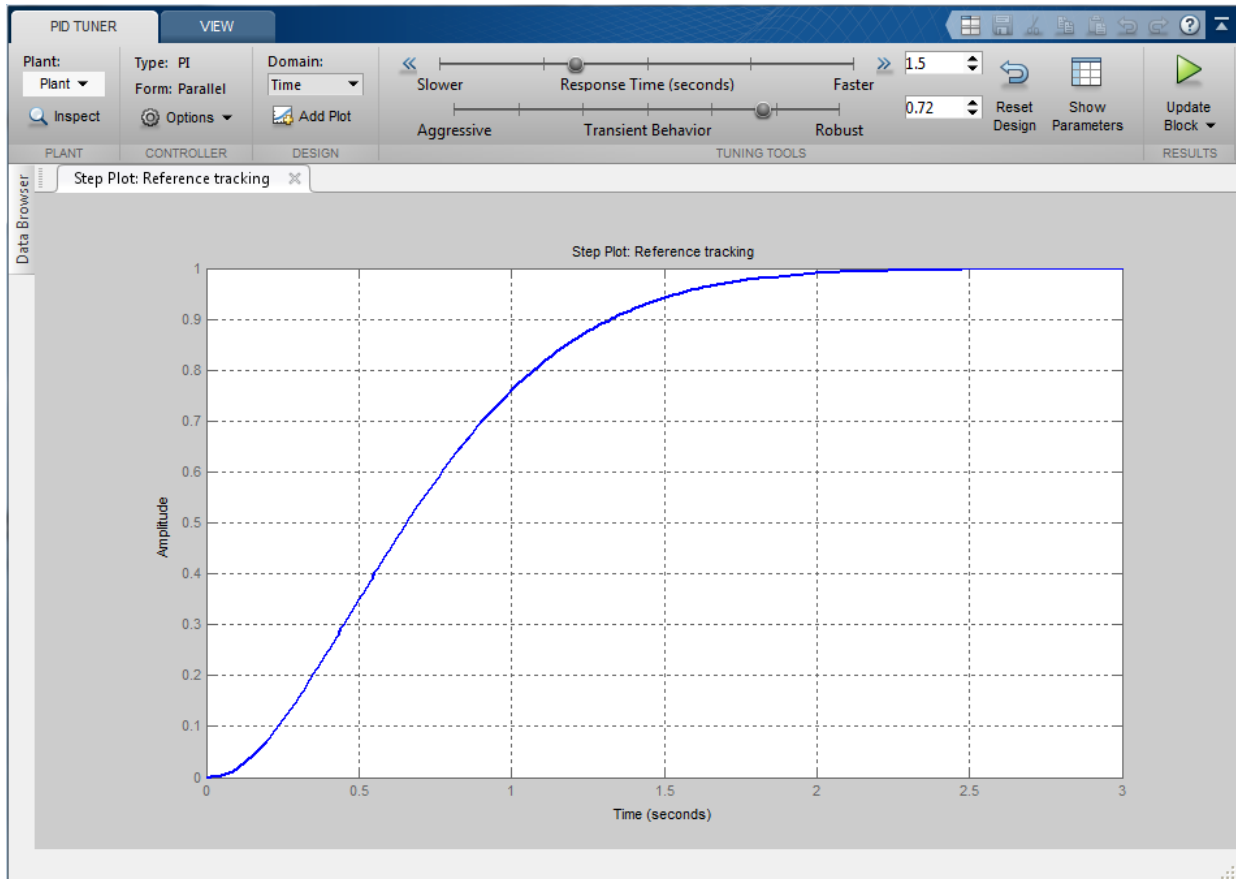
Performance and robustness		
	Tuned	Baseline
Rise time	2.06 seconds	NaN seconds
Settling time	3.45 seconds	NaN seconds
Overshoot	0.401 %	NaN %
Peak	1	Inf
Gain margin	18.9 dB @ 3.27 rad/s	-19.9 dB @ 19 rad/s
Phase margin	69.3 deg @ 0.645 rad/s	-46.6 deg @ 60.3 rad/s
Closed-loop stability	Stable	Undefined

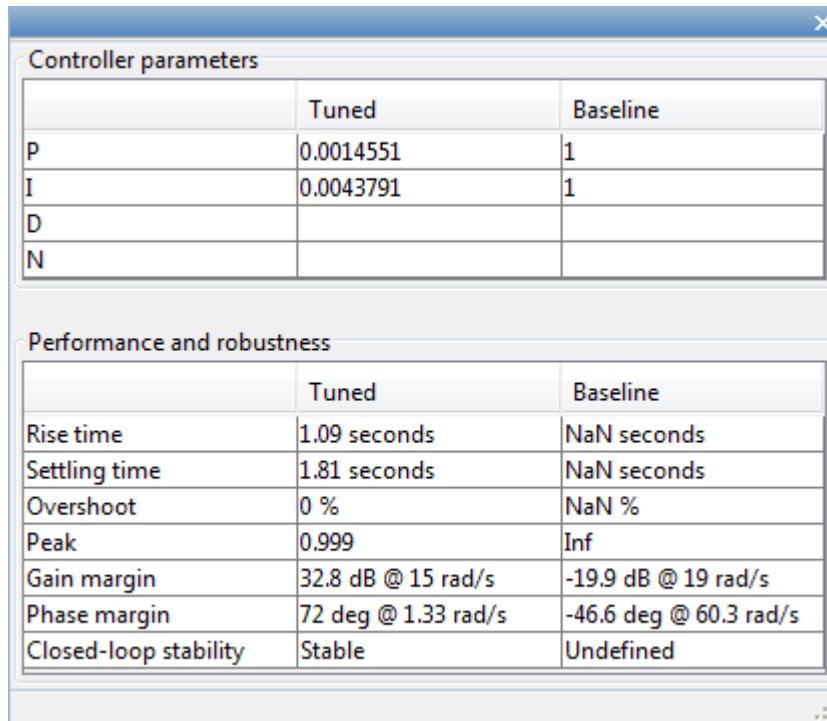
Completing PID Design with Performance Trade-Off

In order to achieve zero overshoot while reducing the settling time below 2 seconds, you need to take advantage of both sliders. You need to make control response faster to reduce the settling time and increase the robustness to reduce the overshoot. For example, you can reduce the response time from 3.4 to 1.5 seconds and increase robustness from 0.6 to 0.72.

The following figure shows the closed-loop response with these settings:

4 PID Control Design





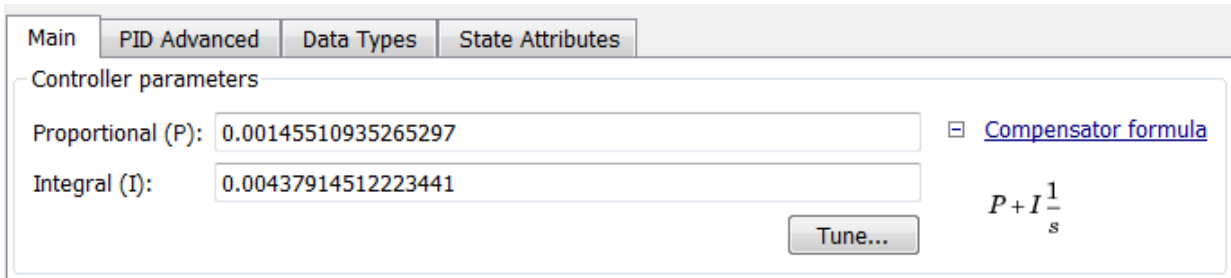
Controller parameters		
	Tuned	Baseline
P	0.0014551	1
I	0.0043791	1
D		
N		

Performance and robustness		
	Tuned	Baseline
Rise time	1.09 seconds	NaN seconds
Settling time	1.81 seconds	NaN seconds
Overshoot	0 %	NaN %
Peak	0.999	Inf
Gain margin	32.8 dB @ 15 rad/s	-19.9 dB @ 19 rad/s
Phase margin	72 deg @ 1.33 rad/s	-46.6 deg @ 60.3 rad/s
Closed-loop stability	Stable	Undefined

Writing the Tuned Parameters to PID Controller Block

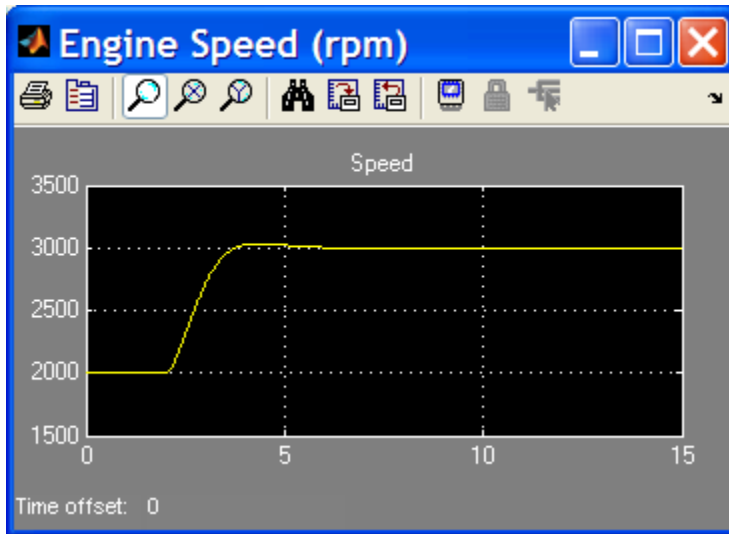
After you are happy with the controller performance on the linear plant model, you can test the design on the nonlinear model. To do this, click **Update Block** in the PID Tuner. This action writes the parameters back to the PID Controller block in the Simulink model.

The following figure shows the updated PID Controller block dialog:



Completed Design

The following figure shows the response of the closed-loop system:



The response shows that the new controller meets all the design requirements.

You can also use the SISO Compensator Design Tool to design the PID Controller block. When the PID Controller block belongs to a multi-loop design task. See the example "Single Loop Feedback/Prefilter Compensator Design".

```
bdclose('scdspeedctrlpidblock')
```